

Subject: Mathematics.

Topic: Angles on parallel lines.

Class: S.S.1.

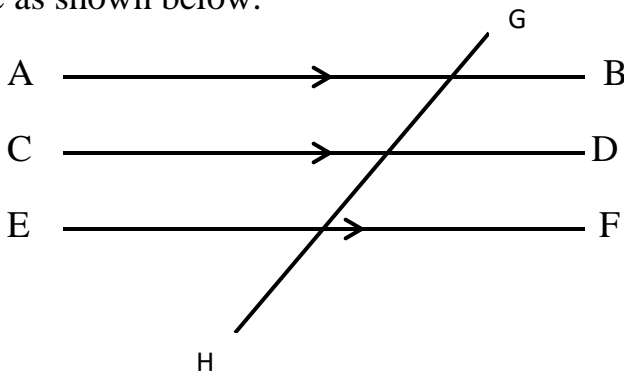
Specific objectives: By the end of this lesson the students should be able to :

- i. State the relationships between the angles on parallel lines.
- ii. Solve simple problems involving angles on parallel lines.

Introduction

It is pertinent to note that parallel lines are lines that are always the same distance apart. They do not meet each other.

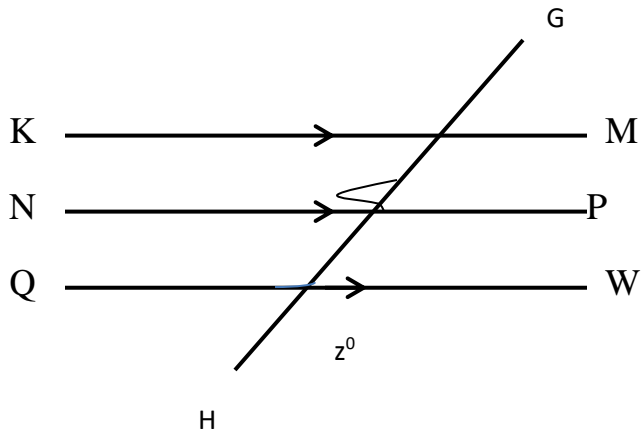
A line that intersects two or more parallel lines is called a transversal. These lines are as shown below.



From the above diagram, lines: AB, CD and EF are parallel while line GH is a transversal. Note that arrows are usually used to indicate parallel lines.

TERMS ASSOCIATED WITH ANGLES ON PARALLEL LINES AND THEIR RELATIONSHIPS

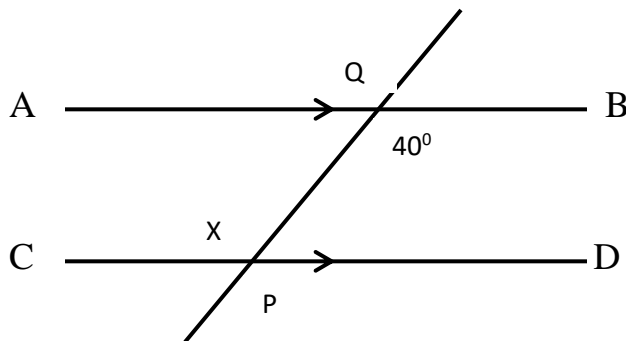
1. **Alternate Angles:** when two angles are formed on opposite sides of the transversal within two parallel lines, such angles are said to be alternate as shown in the diagram below



From the diagram, the angles marked X^0 and r^0 or z^0 and Y^0 are alternate angles respectively. Two alternate angles are equal to each other, hence we have:

i. $X^0 = r^0$ (ii) $Z^0 = y^0$

Example 1: In the diagram below, find the angle marked x.



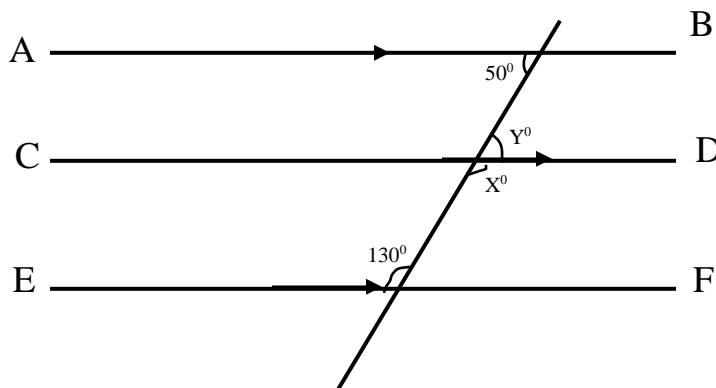
Solution

Angles $\angle PQB$ and $\angle QPC$ are alternate angles and $\angle PQB = 40^0$ $\angle QPC = X$

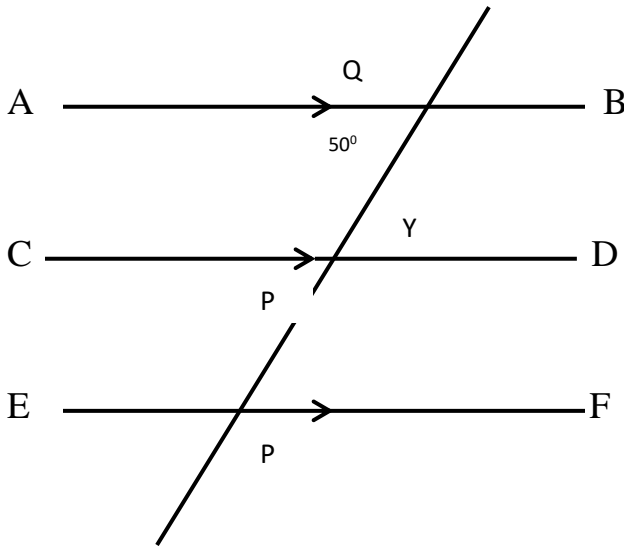
→ $\angle QPC = \angle PQB$ (alternate angles are equal).

Hence, $X = 40^0$

2. Find each of the marked angles in this diagram below



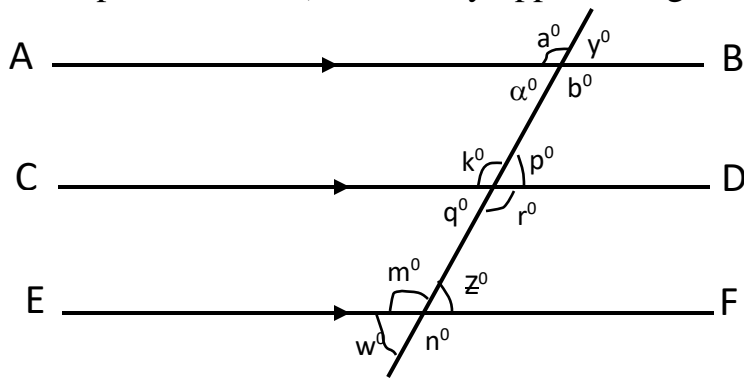
Example 2: In the diagram below, find the angle marked x.



Solution

(i) $y = 50^\circ$ (alternate angles are equal) (ii) $x = 130^\circ$ (same reason)

2. **Vertically Opposite Angles:** two angles are said to be vertically opposite if they are formed opposite each other at the point where the two lines intersect, (whether the two lines are parallel or not). Vertically opposite angles are equal.



From the diagram above, the following pairs of angles are vertically opposite.

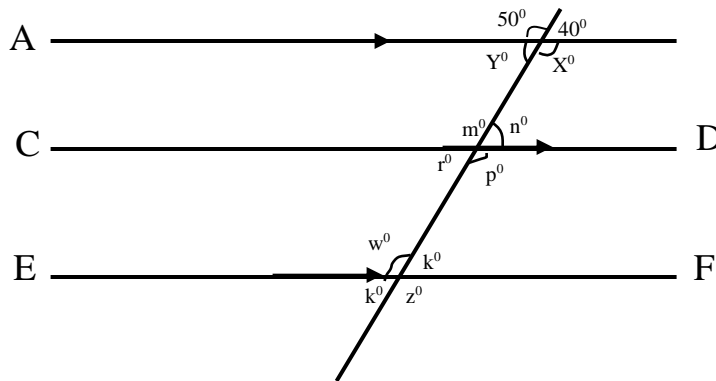
- | | |
|-----------------------------------|----------------------------------|
| (i) a° and b° | (iv) p° and q° |
| (ii) x° and y° | (v) m° and n° |
| (iii) k° and r° | (vi) w° and z° |

since vertically opposite angles are always equal, it follows that:

- | | |
|------------------------------|------------------------------|
| (iv) $a^{\circ} = b^{\circ}$ | (iv) $p^{\circ} = q^{\circ}$ |
| (v) $x^{\circ} = y^{\circ}$ | (v) $m^{\circ} = n^{\circ}$ |
| (vi) $k^{\circ} = r^{\circ}$ | (vi) $w^{\circ} = z^{\circ}$ |

Example:

Find each of the marked angles in the diagram blow.



NB: Always remember that alternate angles are equal and apply it where appropriate.

From the diagram above.

- (i) $x = 50^{\circ}$
 $y = 40^{\circ}$ (Vertically opposite angles are equal)

- (ii) $x = m^{\circ}$
 $y = n^{\circ}$ (alternate angles are equal)

Hence, $m = 50^{\circ}$ and $n = 40^{\circ}$

Also,

- (iii) $m^{\circ} = p^{\circ}$ (Vertically opposite angles are equal)

$$n^{\circ} = r^{\circ}$$

Hence, $p = 50^{\circ}$ and $r = 40^{\circ}$

(iv) $p^{\circ} = w^{\circ}$ (Alternate angles are equal)
 $r^{\circ} = L^{\circ}$

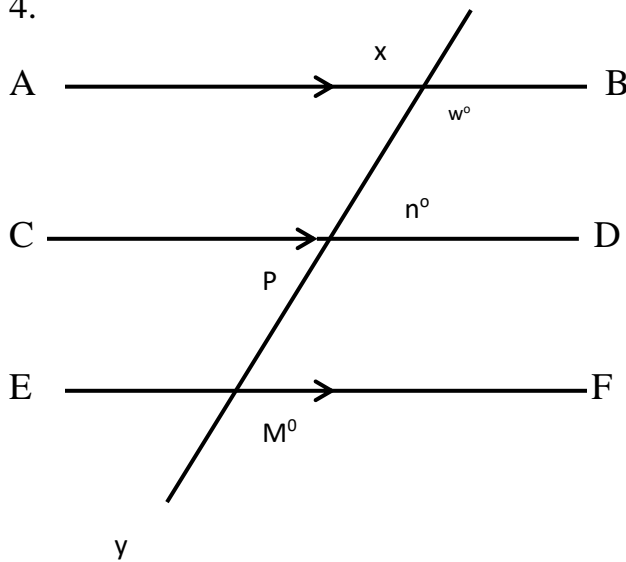
Hence, $w = 50^{\circ}$ and $L = 40^{\circ}$

(v) $w^{\circ} = z^{\circ}$ (Vertically opposite angles are equal)
 $L^{\circ} = k^{\circ}$

Hence, $z = 50^{\circ}$ and $k = 40^{\circ}$

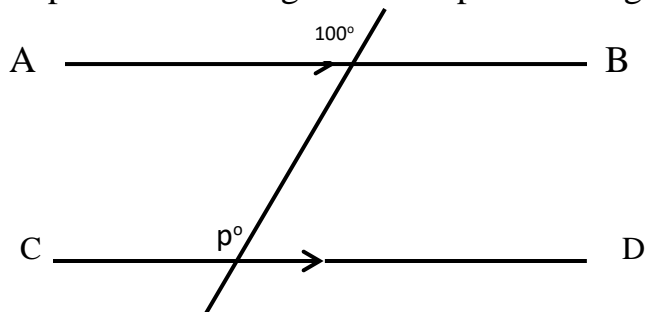
3. **Corresponding Angles:** Angles on a similar position are called corresponding angles and corresponding angles are equal.

4.



Hence, $x^{\circ} = k^{\circ} = y^{\circ}$ and $w^{\circ} = n^{\circ} = m^{\circ}$

Example: Find the angle marked p in the diagram below.

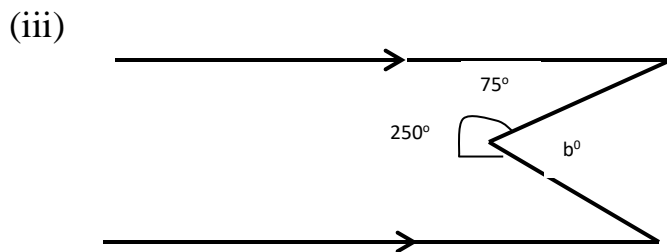
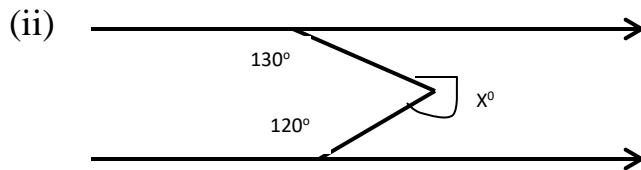
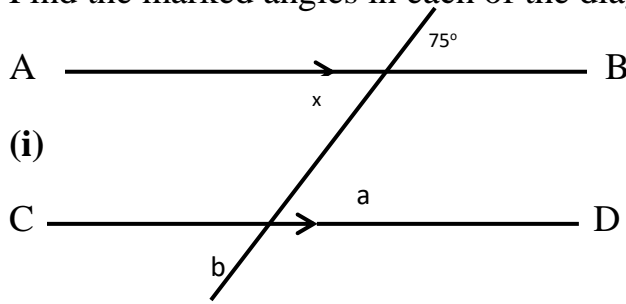


Solution

$P = 100^\circ$ (corresponding angles are equal).

General Examples:

Find the marked angles in each of the diagram below.



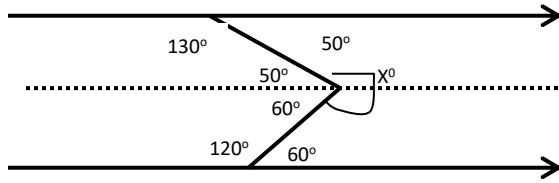
Solution

(i) $x^\circ = 75^\circ$ (Vertically opposite angles)

$x^\circ = a^\circ$ (Alternate angles are equal)

Hence, $a = 75^\circ$ and $b = 75^\circ$

To solve number (ii) we re-draw the diagram, as show below.



Recall the following points:

- (i) Sum of the angles on a straight line is equal to 180°
- (ii) Sum of the angles at a point equals 360°

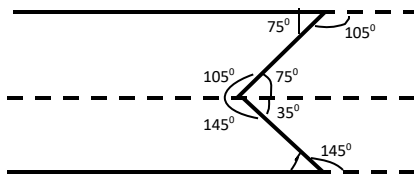
Hence, $50 + 60 + x = 360$ (sum of the angles at a point)

$$110 + x = 360$$

$$x = 360 - 110$$

$$x = 250^{\circ}$$

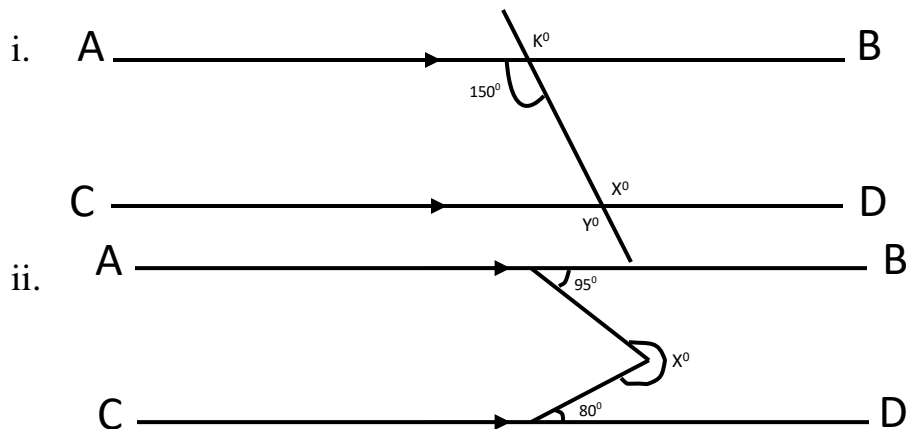
- (iii) Also we re-draw the diagram as shown below.



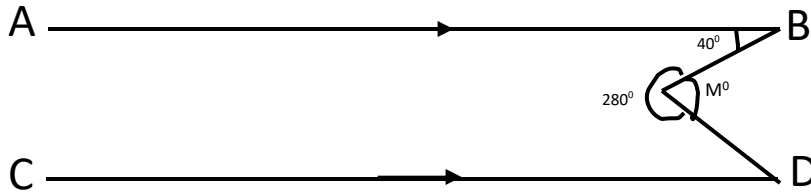
Hence, $b = 35 + 75 = 110^{\circ}$

Evaluation

1. Solve for each of the following marked angles in the diagram below

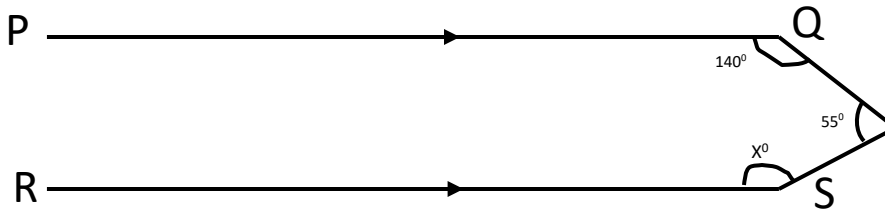


2. Find the angle marked m in the diagram below



Assignment

1. In the diagram lines PQ and RS are parallel. Find X.

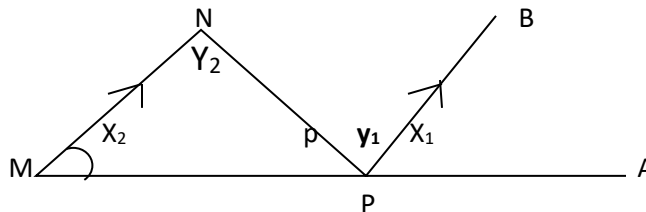


2. Given that $(y-30)^\circ$ and $(50-4y)$ are two corresponding angles. Find the value of y .

TOPIC: Proof of some basic theorems: such as: (i) the sum of the angles of a triangle is 180° (ii) the sum of two opposite interior angles of a triangle is equal to the exterior angle.

THEOREM 1.

The sum of the angles of a triangle is equal to 180° .



Given: any $\triangle MNP$

To prove: $N + M + P = 180^\circ$

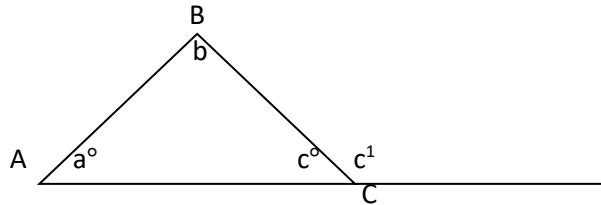
Construction: produce \overline{MP} to a point A

Draw \overline{PB} parallel to \overline{MN}

With the lettering of the fig above
 $Y_2 = Y_1$ (alternate angles)
 $X_2 = X_1$ (corresponding angles)
 $P + y_1 + X_1 = 180$ (sum of angles on a straight line)
 $\Rightarrow P + Y_2 + X_2 = 180$ (Since $X_1 = X_2$; $y_1 = y_2$)
 $\therefore N + M + P = 180^\circ$ (as required)

THEOREM 2

The sum of two opposite interior angles of a triangle is equal to the exterior angle.



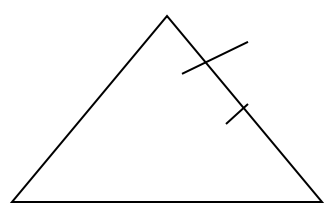
Given: any $\triangle ABC$
 To prove: $\angle ABC + \angle BAC = \angle BCF$
 For convenience: let: $\angle ABC = b^\circ$, $\angle BAC = a^\circ$
 $\angle BCF = c^1$ and $\angle ACB = c$
 With the lettering of the fig. above:
 $\Rightarrow \angle BCF + \angle ACB = 180$ (sum of the angles on a straight line)
 $\Rightarrow c^1 + c = 180^\circ \dots\dots\dots(1)$
 $c = 180 - c^1 \dots\dots\dots(2)$
 Also: $\angle ABC + \angle BAC + \angle ACB = 180^\circ \dots\dots\dots(3)$
 Putting, $180 - c^1$ as c in equation (3) we have:
 $b + a + 180 - c^1 = 180$
 $b + a = 180 - 180 + c^1$
 $\angle ABC + \angle BAC = \angle BCF$ (as required).

It is pertinent to take note of this types of triangles.
 Isosceles triangle: this is a triangle which has two opposite sides equal in length as shown below.

B Vertical angle

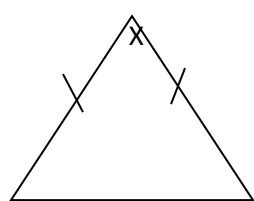
NB: $AB = BC$. Angles A and C are called base angles and they are equal in size (ie $A = C$)
 Simply put: base angles of an isosceles triangle are equal. Angle B is called vertical angle.

EXAMPLES

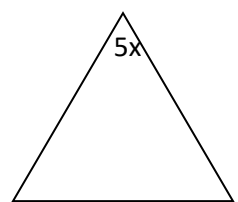


Calculate the marked angles in each of the triangle below.

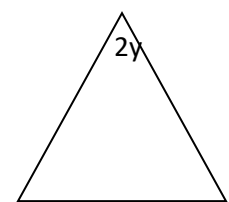
1.



2.



3.



70

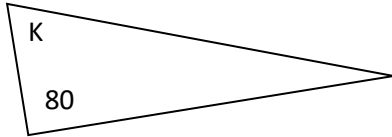
45

50

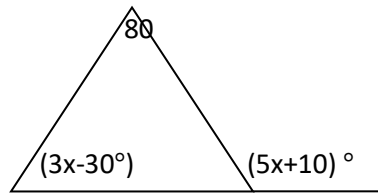
80°

140

4.



5.

**ASSIGNMENT**

1. An isosceles triangle is such that the vertical angle is four times the size of a base angle, calculate the base angle.
2. ABC is an isosceles triangle with its base \overline{BC} produced to point D. if angle (A) = 75° calculate angle C.

MATHEMATICS CLASS: SS1 3RD TERM NOTE

Topic: Polygons

A Polygon is a closed plane figure completely bounded by n straight lines .where $n >_3$

Written below are examples of polygons

Names of polygon	No of sides of the polygon
Triangles	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

ETC....

The following points should be noted about a polygon

- a. A polygon with n –sides, has n – interior angles
- b. A regular polygon is a polygon that has its sides equal and all its angles equal.
Examples of a regular are: 1. an equilateral triangle 2. A square
- c. The sum of the interior angle of a polygon is calculated using the expression below
Sum of interior angles = $(n-2) \times 180^\circ$
- d. Each interior angle = $\frac{(n-2) \times 180}{n}$
- e. sum of the exterior angles is equal to 360°
- f. Each exterior angle = $\frac{360^\circ}{n}$
- g. An interior angle + an exterior angle = 180°

Where n = number of sides

Examples:

1. Find the interior angles of a regular hexagon

Solution

$$\text{Each interior angle} = \frac{(n-2) \times 180}{n}$$

Hexagon has 6 sides = $n = 6$

$$\begin{aligned} \therefore \text{Each interior angle} &= \frac{(6-2) \times 180}{6} \\ &= \frac{4 \times 180}{6} = 120^\circ \end{aligned}$$

2. Find the sum of the interior angles of a polygon with 10 sides

Solution

$$\text{sum of interior } \angle s = (n-2) \times 180$$

$$y-5+y-15+y+y+5+y+15 = (5-2) \times 180$$

$$5y + 5 - 5 = 3 \times 180$$

$$5y + 5 - 5 = 540$$

$$5y = 540 - 5 + 5$$

$$5y = 540$$

$$y = \frac{540}{5}$$

$$y = 108^\circ$$

4. How many sides has a polygon whose interior angles are 135° each

Solutions

$$\text{Each interior angle} = \frac{(n-2) \times 180}{n}$$

$$= 135 = \frac{(n-2) \times 180}{n}$$

$$= (n-2) \times 180 = 135n$$

$$180n - 360 = 135n$$

$$180n - 135n = 360$$

$$45n = 360$$

$$N = \frac{360}{45}$$

$$N = 8 \text{ sides}$$

5. How many sides polygon whose sum of interior angles is 1080°

Solution

$$\text{Sum of interior angle} = (n-2) \times 180$$

$$1080 = (n-2) \times 180$$

$$N-2 = \frac{1080}{180}$$

$$N-2 = 6$$

$$n = 6 + 2$$

$$n = 8$$

6. The exterior angles of polygon are = $4x$, $3x$, x , $4x$, $3x$, and $5x$. FIND X

Solution

$$\text{Sum of exterior angles} = 360$$

$$= 4x + 3x + x + 4x + 3x + 5x = 360$$

$$20x = 360$$

$$X = \frac{360}{20}$$

$$X = 18^\circ$$

How many sides has a polygon whose exterior angles are 60° each

Solution

$$\text{Each exterior angles} = \frac{360}{n}$$

$$= 60 = \frac{360}{n}$$

$$N = \frac{360}{60} = \frac{36}{6} = 6 \text{ sides}$$

Assignment

1. Given that the angles of a polygon are $4x$, 100 , 110 , $(2x + 6)$ and $3x$. find x
2. One interior angles of a hexagon is 150° and each of the remaining interior angles is equal to y . find y
3. In an octagon, b five of the angles are equal and each of the other three angles is 20° greater than each of five other ones. Find the angles of the octagon.

SUBJECT: MATHEMATICS

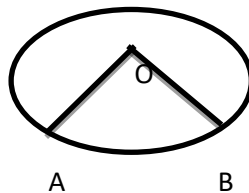
CLASS: SS1

WEEK 4 NOTE

ARC AND SECTOR OF A CIRCLE

Arc of a circle and its length (L)

Consider the circle below with center (o) and radii OA and OB



From the circle above; the portion (AB) of the circumference of the circle cut-off by two radii OA and OB is called an arc of a circle.

If AB represent the length (L) of the arc and the angle which the arc subtends at the center of the circle is θ° , then the formula below is used to calculate the length of any arc of a circle:

$$\text{Length of an arc (L)} = \frac{\theta}{360} \times 2\pi r$$

Where: r = radius of the circle

π = a constant, usually take as $\frac{22}{7}$

θ = angle which the arc subtends at the center of the circle.

EXAMPLES

1. Calculate the length of an arc of a circle of radius 14cm which subtends an angle 120° at the center of the circle. (Take; $\pi = \frac{22}{7}$)

Solution

$$L = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{120 \times 2 \times 22 \times 14}{360 \times 7}$$

$$L = \frac{2 \times 22 \times 2}{3}$$

$$= 29.3\text{cm}$$

2. An arc of a circle is 17.6cm long, if the angle subtends at the center of the circle by the arc is 72° . Calculate the radius of the circle. ($\pi = \frac{22}{7}$).

Solution

$$L = \frac{\theta}{360} \times 2\pi r$$

$$17.6 = \frac{72}{360} \times 2 \times \frac{22}{7} \times r$$

$$\text{Hence, } r = \frac{17.6 \times 2520}{3168} = 14\text{cm}$$

3. Calculate the angle subtends by an arc of length 19.8cm at the center of a circle of radius 10.5cm. ($\pi = \frac{22}{7}$).

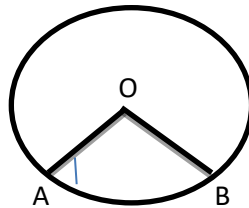
Solution

$$L = \frac{\theta}{360} \times 2\pi r$$

$$19.8 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 10.5$$

$$\text{Hence, } \theta = \frac{19.8 \times 2520}{462} = 108^\circ$$

Sector of a Circle: A sector of a circle is the portion of the circle cut-off by two radii and an arc of two such radii, as shown below.



We shall now see the formulae with which we can calculate the: (i) area of the sector (ii) perimeter of the sector.

$$1. \text{ Area of a sector} = \frac{\theta}{360} \times \pi r^2$$

$$2. \text{ Perimeter of a sector} = 2r + \text{length of an arc}$$

$$= 2r + L$$

$$= 2r + \frac{\theta}{360} \times 2\pi r$$

Examples

1. Calculate the area of a sector of a circle of radius 9cm if the angle of the sector at the center is 150° ($\pi = \frac{22}{7}$)

Solution:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{150 \times 22 \times 9^2}{360 \times 7}$$

$$= 106.07\text{cm}^2$$

2. Calculate the (i) area (ii) perimeter of a sector of a circle of radius 10cm which subtends angle 108° at the center of the circle. ($\pi = \frac{22}{7}$)

Solution:

$$\text{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{108 \times 22 \times 10^2}{360 \times 7}$$

$$= 94.3\text{cm}^2$$

$$\text{Perimeter of a sector} = 2r + \frac{\theta}{360} \times 2\pi r$$

$$\begin{aligned}
&= 2 \times 10 + \frac{108}{360} \times 2 \times \frac{22}{7} \times 10 \\
&= 20 + 18.9 \\
&= 38.9 \text{ cm}
\end{aligned}$$

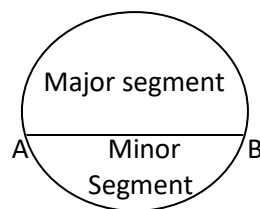
ASSIGNMENT

1. An arc of a circle of radius 12cm subtends angle 106° at the center of a circle. Calculate the (i) the length of the arc (ii) perimeter of the sector (iii) area of a the sector. ($\pi = \frac{22}{7}$)
2. The area of a sector of a circle of radius 9cm is 64cm^2 , calculate the angle of the sector, to the nearest degree.

WEEK 5

SEGMENT OF A CIRCLE

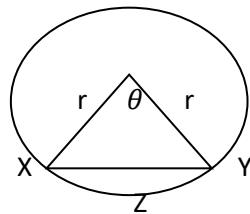
Consider the diagram below, a circle with center (o) has been divided into two portions, called segments.



The bigger portion is called major segment, while the smaller portion is called minor segment. We shall concern ourselves with (i) the area of a segment and (ii) perimeter of a segment of a circle.

(i) Area of a segment of a circle:

The diagram below is a circle with center (o) and radius r . The angle at the center of the circle is θ° .



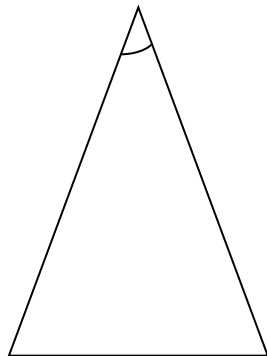
The area of the segment (or the shaded portion) calculated by subtracting the area of triangle OXY from the area of the sector OXZY.

$$\gg \text{Area of the segment (A)} = \text{area of sector (OXZY)} - \text{area of triangle(OXY)}$$

But

$$\text{Area of a sector of a circle} = \frac{\theta}{360} \times \pi r^2$$

And area of the triangle OXY is obtained as shown below



Area of $\triangle XOY = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times OY \times XW$$

$$= \frac{1}{2} \times r \times h \dots (1)$$

$$\text{But } \sin\theta = \frac{h}{r}$$

$$\Rightarrow h = r \sin\theta \dots (2)$$

Putting eqn(2) in eqn(1); we have that

$$\text{Area of } \triangle XOY = \frac{1}{2} \times r \times r \sin\theta$$

$$= \frac{r^2 \sin\theta}{2}$$

$$\text{Area of segment} = \frac{\theta}{360} \times \pi r^2 - \frac{r^2 \sin\theta}{2}$$

$$= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

Examples

1. Calculate the area of a segment of a circle of radius 9cm if the angle at the center of the circle is 108°

$$\left(\pi = \frac{22}{7} \right)$$

Solution

$$\text{Area} = r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

$$= 9^2 \left[\frac{22 \times 108}{7 \times 360} - \frac{\sin 108}{2} \right]$$

$$= 81 \left[\frac{2376}{2520} - \frac{0.9511}{2} \right]$$

$$= 81 \times 0.4673$$

$$\text{Area of segment} = 37.85 \text{cm}^2$$

2. Calculate the area of a segment of a circle of radius 12cm if the angle at the center of the circle is 150° . $\left(\pi = \frac{22}{7} \right)$

Solution

$$\text{Area of segment} = r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

$$= 12^2 \left[\frac{22 \times 150}{7 \times 360} - \frac{\sin 150}{2} \right]$$

$$= 144 \left[\frac{3300}{2520} - \frac{0.5}{2} \right]$$

$$= 144 \times 1.0595$$

$$= 152.57 \text{cm}^2$$

3. The diagram below show a portion of a circle, calculate the area of the shaded portion, if $\angle O = 90^\circ$ $\left(\pi = \frac{22}{7} \right)$.

Solution

$$\text{Area of the shaded portion} = r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

$$= 10^2 \left[\frac{22 \times 90}{7 \times 360} - \frac{\sin 90}{2} \right]$$

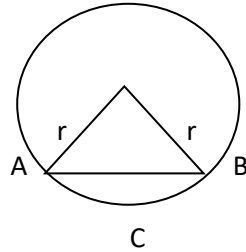
$$= 100 \left[\frac{1980}{2520} - \frac{1}{2} \right]$$

$$= 100 \times 0.2857$$

$$= 28.57 \text{cm}^2$$

PERIMETER OF A SEGMENT OF A CIRCLE

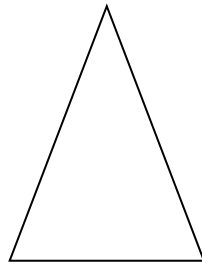
The perimeter of a segment of a circle is the distance round the segment. This distance consists of the length of a chord (AB) and the length of an arc (ACB) that formed the segment as shown below.



Perimeter of the segment (the shaded portion) = length of chord (AB) + length of an arc (ACB)
But

$$\text{Length of an arc} = \frac{\theta}{360} \times 2\pi r \dots (1)$$

Then the length of the chord is obtained as shown below consider $\triangle AOB$.



Point N is the midpoint of AB, hence $AN = NB$ and $\angle AOB$ is bisected so that we have $(\frac{\theta}{2})^\circ$ each. Using $\triangle AON$, a right angled \triangle at N then; $\frac{\sin \theta}{2} = \frac{AN}{r}$

$$AN = r \sin \frac{\theta}{2}$$

$$\text{Also } NB = r \sin \frac{\theta}{2} \text{ (since } AN = NB)$$

$$AB = AN + NB$$

$$= r \sin \frac{\theta}{2} + r \sin \frac{\theta}{2}$$

$$\text{Length of chord (AB)} = 2r \sin \frac{\theta}{2} \dots (2)$$

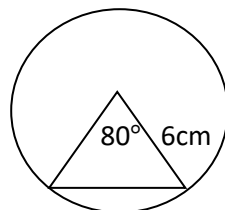
$$\gg \text{ Perimeter of a segment} = 2r \sin \frac{\theta}{2} + \frac{\theta}{360} \times 2\pi r$$

$$= 2r \left[\sin \frac{\theta}{2} + \frac{\pi \theta}{360} \right]$$

Examples

1. find the perimeter of a segment of a circle of radius 6cm if the angle at the centre of the circle is 80° .
($\pi = \frac{22}{7}$).

Solution



$$\begin{aligned} \text{Perimeter of the segment} &= 2r \left[\sin \frac{\theta}{2} + \frac{\pi\theta}{360} \right] \\ &= 2 \times 6 \left[\sin \frac{80}{2} + \frac{22 \times 80}{7 \times 360} \right] \\ &= 12 \left[\sin 40 + \frac{1760}{2520} \right] \\ &= 12 \times 0.76268 \\ &= 9.15 \text{ cm} \end{aligned}$$

2. Calculate the perimeter of a segment of a circle radius 16cm if the angle at the center of the circle is 170° ($\pi = \frac{22}{7}$)

Solution

$$\begin{aligned} \text{Perimeter} &= 2r \left[\sin \frac{\theta}{2} + \frac{\pi\theta}{360} \right] \\ &= 2 \times 16 \left[\sin \frac{170}{2} + \frac{22 \times 170}{7 \times 360} \right] \\ &= 32 \left[\sin 85 + \frac{3740}{2520} \right] \\ &= 32 \times 2.4803 \\ &= 79.4 \text{ cm} \end{aligned}$$

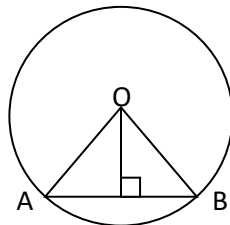
Assignment

Calculate the: (i) area (ii) perimeter of a segment of a circle of Radius 7cm if the angle at the center of the circle is 270° . ($\pi = \frac{22}{7}$).

WEEK 8

CHORD OF A CIRCLE

Consider the circle below a radius, OA and OB, with center, O.



From the circle above, a line AB, drawn from one point of the circumference (A) to another point (B) is called a chord of circle.

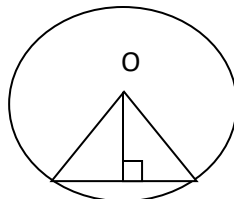
The following points should be carefully noted about a chord.

1. Line (AB) is called the length of a chord
2. Line (ON) divides the length of the chord (AB) into two equal parts, such that: $\angle AN = \angle NB$ & $\angle AB = \angle AN + \angle NB$
3. Line (ON) is called the distance of the chord from the center of the circle, (O)
4. Line (OA) = line (OB); i.e. $\angle OA = \angle OB$ (radii of a circle)

Worked examples

1. A chord of a circle of radius 5cm is 6cm long, how far is the chord from the center of the circle?

Solution



5cm
A B

Using Pythagoras theorem

$$OA^2 = OB^2 + AB^2$$

$$5^2 = X^2 + 3^2$$

$$25 = X^2 + 9$$

$$\gg X^2 = 25 - 9$$

$$X^2 = 16$$

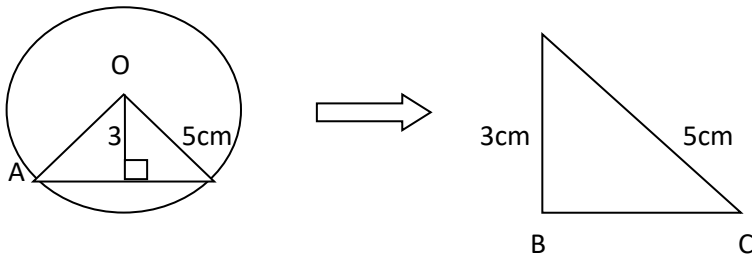
$$X = \sqrt{16}$$

$$X = 4\text{cm}$$

The chord is 4cm away from the center of the circle.

2. A chord is drawn 3cm away from the center of a circle of radius 5cm. calculate the length of the chord.

Solution



$$5^2 = 3^2 + BC^2$$

$$25 = 9 + BC^2$$

$$BC^2 = 16$$

$$BC = \sqrt{16} = 4$$

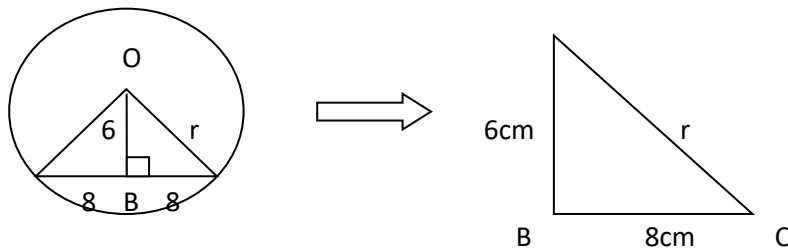
\gg The length of a chord $AC = AB + BC$

$$AC = 4 + 4$$

$$= 8\text{cm}$$

3. A chord of a circle of length 16cm is 6cm from the center of the circle, calculate the radius of the circle.

Solution



$$r^2 = 6^2 + 8^2$$

$$r^2 = 36 + 64$$

$$r^2 = 100$$

$$r = \sqrt{100}$$

$r = 10$.

∴ The radius of the circle is 10cm

4. A chord 30cm long is 20cm from the center of a circle. Calculate the radius of the chord.

Solution

$$r^2 = 20^2 + 15^2$$

$$= 400 + 225$$

$$= 625$$

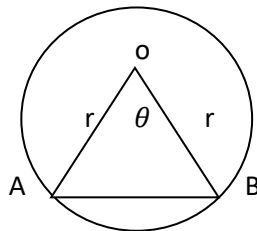
$$= \sqrt{625}$$

$$= 25.$$

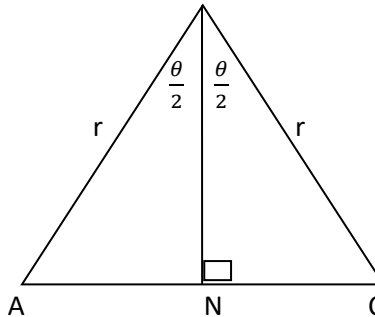
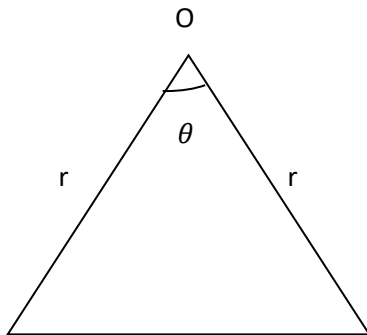
∴ The radius of the circle is 25cm

LENGTH OF A CHORD WHICH SUBTENDS AN ANGLE AT THE CENTER OF A CIRCLE

Consider a chord (AB) of a circle of radius (r) which subtends angle, θ at the center of the circle, as shown below.



The formula with which to calculate the length of the chord is derived as shown below.



Note: Line /ON/ bisects angle AOB into two equal parts, hence, line /AC/ is also bisected by line ON with point (N) as the midpoint of line /AC/. Hence, /AN/= /NC/

$$\therefore /AC/ = /AN/ + /NC/$$

Using trigonometric ratio: $\gg \sin \frac{\theta}{2} = \frac{NC}{OC}$

$$\gg \sin \frac{\theta}{2} = \frac{NC}{r} \rightarrow NC = r \sin \frac{\theta}{2}$$

$$/AC/ = 2r \sin \frac{\theta}{2}$$

$$\therefore \text{Length of a chord (AB)} = 2r \sin \frac{\theta}{2}$$

Examples

1. Calculate the length of a chord of a circle of radius 10cm if the angle at the center of the circle is 60°

Solution

$$\text{Length of the chord} = 2r \sin \frac{\theta}{2}$$

$$= 2 \times 10 \times \sin \frac{60}{2}$$

$$= 2 \times 10 \times \sin 30$$

$$= 2 \times 10 \times 0.5$$

$$= 10\text{cm}$$

2. A chord of a circle of radius 15cm subtends an angle of 80° at the center of the circle. Calculate the length of the chord.

Solution

$$\text{Length of a chord} = 2r \sin \frac{\theta}{2}$$

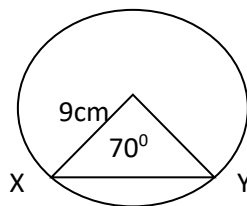
$$= 2 \times 15 \times \sin \frac{80}{2}$$

$$= 2 \times 15 \times \sin 40$$

$$= 2 \times 15 \times 0.6428$$

$$= 19.28\text{cm.}$$

3. Find the length XY in the diagram below.



Solution

$$XY = 2r \sin \frac{\theta}{2}$$

$$= 2 \times 9 \times \sin \frac{70}{2}$$

$$= 2 \times 9 \times \sin 35$$

$$= 18 \times 0.5736$$

$$XY = 10.33\text{cm}$$

ASSIGNMENT

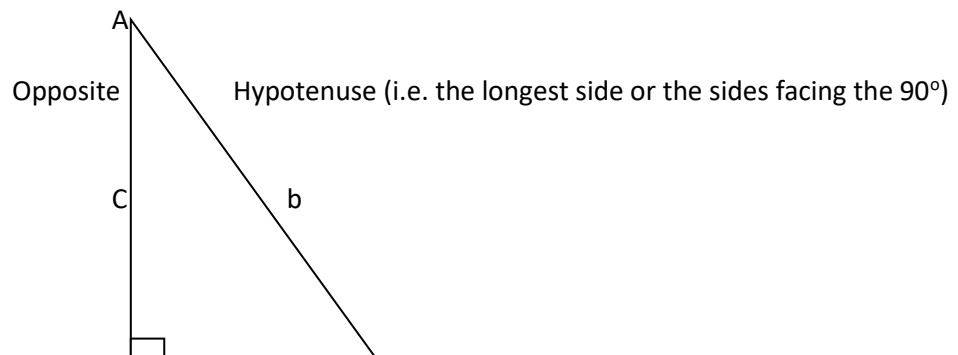
1. Calculate the length of a chord of a circle of radius 13cm if the chord is 12cm from the center of the circle.
2. A chord of length 30cm is 8cm from the center of a circle of radius (r). Calculate the value of r
3. Calculate the length of a chord of a circle of radius 8cm if the angle at the center of the circle is 75° .

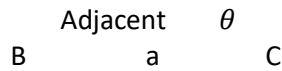
WEEK 6

TRIGONOMETRIC RATIOS AND PYTHAGORAS THEOREM

Trigonometric ratios:

Consider a right-angled triangle, ABC, where A, B, C are the angles and the sides facing angles a, b, and c respectively as shown below.





It is the pertinent to note that the trigonometric ratios. Can be stated as follows:

$$1. \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{c}{b}$$

$$2. \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{b}$$

$$3. \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{a}$$

The first three (3) trigonometric ratios are the most commonly used, hence should be carefully noted.

They can be easily remembered using the word, "SOHCAHTOA".

However, other trigonometric ratios you have to study are:

$$4. \text{Cosec } \theta = \frac{1}{\sin \theta}$$

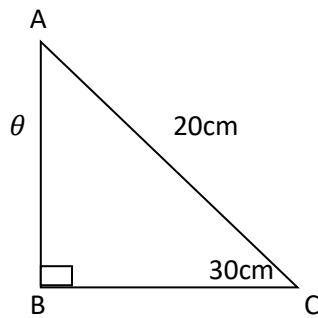
$$5. \text{Sec } \theta = \frac{1}{\cos \theta}$$

$$6. \text{Cot } \theta = \frac{1}{\tan \theta}$$

These cart three trigonometric ratios should also be well noted.

Examples

1. Calculate the side marked x , in the diagram below.



Solution

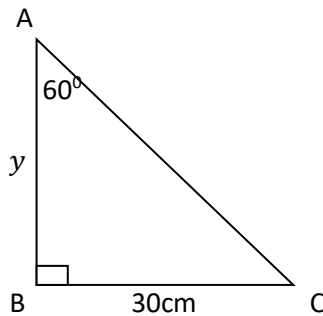
$$\text{Using: } \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\gg \sin 30 = \frac{x}{20}$$

$$= \frac{0.5}{1} = \frac{x}{20}$$

$$\gg x = 10\text{cm.}$$

2. Find the side marked y in the diagram below.



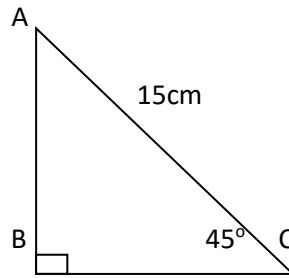
Solution

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 60 = \frac{30}{y}$$

$$y = \frac{30}{\tan 60} = \frac{30}{1.7321} = 17.3 \text{ cm}$$

3. What is the value of k in the diagram below



Solution

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

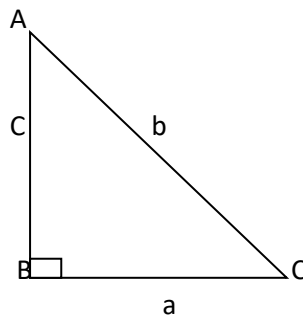
$$\cos 45 = \frac{k}{15}$$

$$k = 15 \times \cos 45$$

$$15 \times 0.7071 = 10.6 \text{ cm.}$$

PYTHAGORAS THEOREM

Consider a right-angled triangle below.



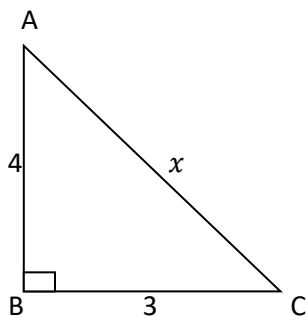
The Pythagoras theorem succinctly states that the square of the hypotenuse of a right-angled is equal to the sum of the squares of the other two sides. Using the triangle below; we have:

$$(\text{Hypotenuse})^2 = (\text{side AB})^2 + (\text{side BC})^2$$

$$\gg b^2 = c^2 + a^2 \rightarrow \text{Pythagoras theorem.}$$

Examples

1. In the diagram below, find the side marked x

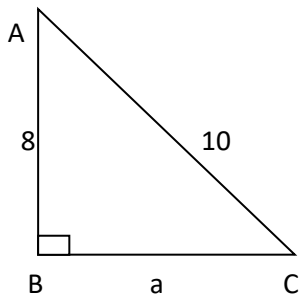


Solution

$$b^2 = c^2 + a^2$$

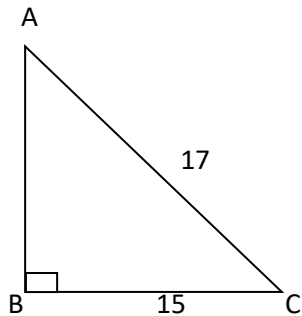
$$\begin{aligned}
 x^2 &= 4^2 + 3^2 \\
 &= 16 + 9 \\
 &= 25 \\
 &= \sqrt{25} = 5
 \end{aligned}$$

2. What is the side marked a, in the diagram below.



$$\begin{aligned}
 b^2 &= c^2 + a^2 \\
 10^2 &= 8^2 + a^2 \\
 100 &= 64 + a^2 \\
 &>> a^2 = 100 - 64 \\
 a^2 &= 36 \\
 a &= \sqrt{36} = 6
 \end{aligned}$$

3. Given the triangle below, find the unknown side



Solution

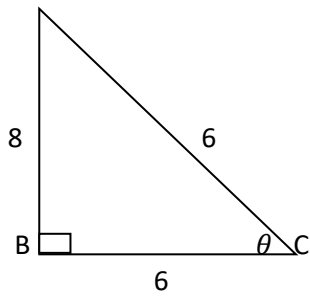
$$\begin{aligned}
 b^2 &= a^2 + c^2 \\
 17^2 &= 15^2 + c^2 \\
 289 &= 225 + c^2 \\
 c^2 &= 289 - 225 \\
 c^2 &= 64 >> c = \sqrt{64} = 8
 \end{aligned}$$

3. Given that $\tan \theta = \frac{8}{6}$, evaluate: (i) $\sin \theta$ (ii) $\cos \theta$ (iii) $\frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta}$

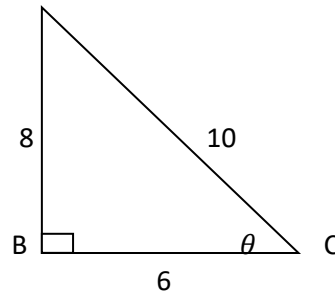
Solution

A

A



→



By using Pythagoras theorem: $b^2 = 8^2 + 6^2$

$$= 64 + 36$$

$$b^2 = 100 \gg b = \sqrt{100}$$

$$b = 10$$

$$(i) \sin \theta = \frac{\text{opp}}{\text{hyp}} \rightarrow \sin \theta = \frac{8}{10} = \frac{4}{5}$$

$$(ii) \cos \theta = \frac{\text{adj}}{\text{hyp}} \rightarrow \cos \theta = \frac{6}{10} = \frac{3}{5}$$

$$(iii) \frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta} = \left(\frac{4}{5} + \frac{3}{5} \right) \div \left(\frac{3}{5} - \frac{4}{5} \right)$$

$$= \left(\frac{4+3}{5} \right) \div \left(\frac{3-4}{5} \right)$$

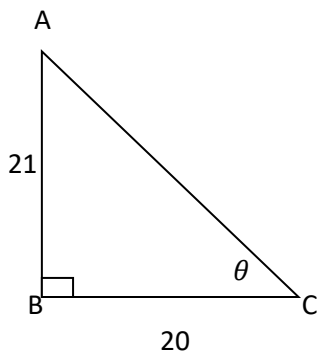
$$= \frac{7}{5} \div \frac{5}{-1}$$

$$= \frac{7}{5} \times \frac{5}{-1}$$

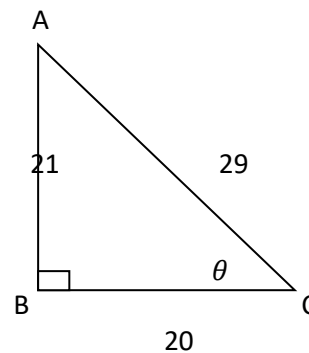
$$= -7.$$

4. If $\tan \theta = \frac{21}{20}$, evaluate (i) $1 + \sin \theta$ (ii) $\sec \theta$ (iii) $\cot \theta$ (iv) $\text{cosec } \theta$ (v) $1 - \cos \theta$

Solution



→



$$b^2 = 21^2 + 20^2 = 441 + 400$$

$$b = \sqrt{841} = 29$$

$$(i) \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{21}{29}$$

$$\gg 1 + \sin^2 \theta = 1 + \left(\frac{21}{29} \right)^2 = 1 + \frac{441}{841} = \frac{841+441}{841} = \frac{1282}{841} = 1.52$$

$$(ii) \sec \theta = \frac{1}{\cos \theta}; \text{ but } \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{20}{29}$$

$$\gg \sec \theta = \frac{1}{20/29} = \frac{29}{20}$$

$$(iii) \cot \theta = \frac{1}{\tan \theta} = \frac{1}{21/20} = \frac{20}{21}$$

$$(iv) \operatorname{Cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{21/29} = \frac{29}{21}$$

$$(v) 1 - \cos \theta \\ = 1 - \frac{20}{29} = \frac{29-20}{29} = \frac{9}{29}$$

ASSIGNMENT

1. Given that $\tan \theta = \frac{5}{12}$, evaluate $\sin \theta + \cos \theta$

2. If x is an acute angle and $\tan x = \frac{3}{4}$, evaluate $\frac{\cos x - \sin x}{\cos x + \sin x}$